AMERICAN UINVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
EECE 460 SPRING 2004-2005
Control Systems
Quiz I SOLUTION

Name:
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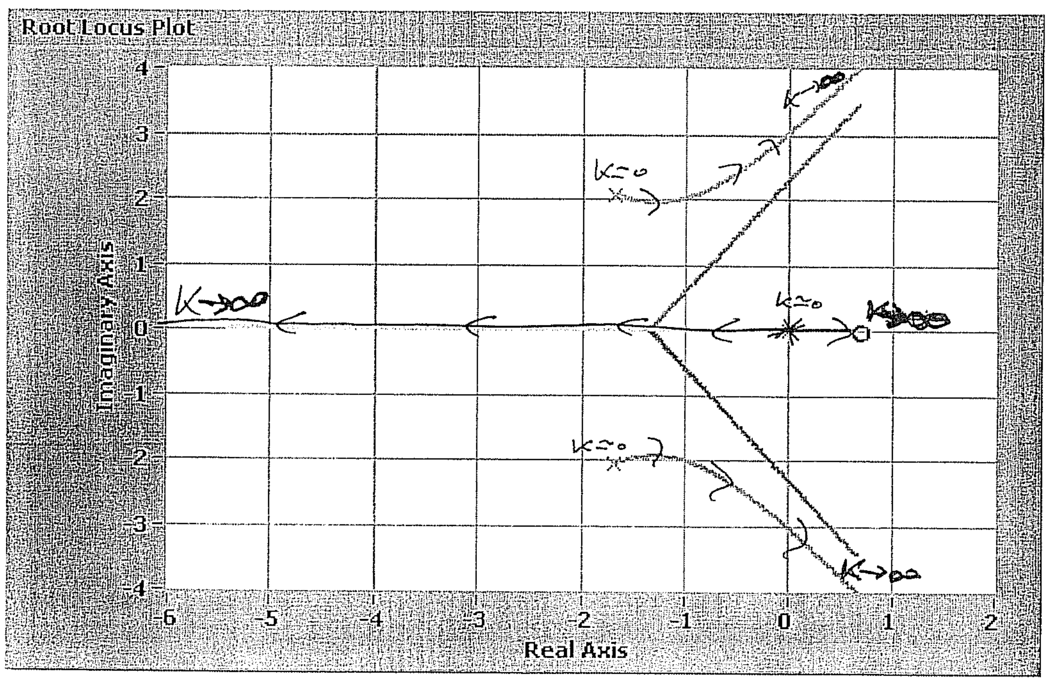
1.5 hours. Total of 100 points
March 22, Open Book Exam, 2 pages
YOU MUST RETURN THIS EXAM WITH YOUR ANSWER BOOKLET

Problem 1: (50 pts)

Consider the system whose open-loop transfer function $G(s)H(s)$ (K is positive gain varying from 0 to infinity) is given by:

$$G(s)H(s) = \frac{K(s-0.6667)}{s^4 + 3.3401s^3 + 7.0325s^2}$$



+5 1. Supply open loop transfer function poles and zeros P: (0, 0, -1.7 ± j2) Z: (0.66

+10 2. The plot shows an incomplete root locus, and it includes the asymptotes:
   a. Complete the root locus (real axis part)
   b. Show on plot values of K (0 and infinity)
   c. Direction arrows of locus as K is increased

+5 3. Based on the plot, for what values of K does the system have a double pole? K= c
+7 4. Characterize the stability of the open loop transfer function Marginally Stable
+8 5. Characterize the stability of the closed loop transfer function Unstable
+8 6. Design if possible the K gain (Proportional controller) in order to have a step response of the closed loop transfer function Maximum overshoot less tan 10% and settle time less than 3 sec. Impossible, Unstable
+7 7. Design if possible the gain K in order to obtain a static position steady state error constant equal to 10. Impossible, Unstable

Problem 2: (50 pts)

A regulator system has a plant

$$\frac{Y(s)}{U(s)} = \frac{10}{(s+1)(s+2)(s+3)}$$

Define state variables as

$$x_1 = y \qquad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$x_2 = \dot{x}_1$$

$$x_3 = \dot{x}_2 \qquad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad D = [0]$$

+10 1. Derive the corresponding state model (A, B, C, D matrices)
+5 2. Based on assigning the states as given, is the derived state model unique? Why. Yes ... are defin...
+6 3. Find the poles of the system. $-1, -2, -3$
+6 4. Find the eigenvalues of the corresponding matrix A of the state model. $-1, -2, -3$ (S...
+5 5. Is the system stable? Why. Yes; ∀ poles ∈ L.H.P.
+8 6. Design if possible the gain K (1x3) vector for the input U(t) = -KX(t) to place the closed-loop X(t) = (A-BK)X(t) poles at desired locations:

$K = [15.4, 4.5, 0.8]$

$$s = -2 + j2\sqrt{3}, \quad s = -2 - j2\sqrt{3}, \quad s = -10$$

+10 7. For the achieved poles of the controlled system of the part (6), what are the corresponding delivered step response specifications (only supply settle time and maximum overshoot).

$S_{1,2} = -2 \pm j2\sqrt{3}$ are Dominant Poles

$(s - s_1)(s - s_2) =$

$= s^2 + 2\zeta\omega_n s + \omega_n^2$

$\Rightarrow \boxed{\omega_n = 4 \quad \zeta = 0.5}$

$\Rightarrow \boxed{M_p = 16\,\% \quad t_{s\,2\%} = 2 \text{ sec}}$